P.3

Linear Equations and Inequalities

Equations

An **equation** is a statement of equality between two expressions. Here are some properties of equality that we use to solve equations algebraically.

Properties of Equality

Let *u*, *v, w*, and *z* be real numbers, variables, or algebraic expressions.

1. Reflexive — $u = u$
2. Symmetric — If $u = v$, then $v = u$.
3. Transitive — If $u = v$, and $v = w$, then $u = w$.
4. Addition — If $u = v$, and $w = z$, then $u + w = v + z$.
5. Multiplication — If $u = v$ and $w = z$, the $uw = vz$.

Solving Equations

A **solution of an equation in x** is a value of x for which the equation is true. To **solve an equation in x** means to find all values of x for which the equation is true, that is, to find all solutions of the equation.

Ex. Confirming a Solution

Prove that x = -2 is a solution of the equation $x^3 - x + 6 = 0$.

Substituting -2 in for x we have

$$(-2)^3 - (-2) + 6 = 0$$

$$-8 + 2 + 6 = 0$$

$0 = 0$ *is true* therefore, -2 is a solution.

Linear Equations in One Variable

The most basic equation in algebra is a *linear equation*.

Linear Equation in x

A linear equation in x is one that can be written in the form $ax + b = 0$, where a and b are real numbers with $a \neq 0$.

Equivalent: two or more equations are equivalent if they have the same solutions.

Operations for Equivalent Equations

An equivalent equation is obtained if one or more of the following operations are performed.

1. Combine like terms, reduce fractions, and remove grouping symbols.
2. Perform the same operation on both sides. (add, subtract, multiply, or divide)

Linear Inequalities in x

A linear inequality in x is one that can be written in the form $ax + b < 0$, $ax + b \leq 0$, $ax + b > 0$, and $ax + b \geq 0$, where a and b are real numbers with $a \neq 0$.

Properties of Inequalities

Let u, v, w, and z be real numbers, variables, or algebraic expressions, and c a real number.

1. Transitive — If $u < v$ and $v < w$, then $u < w$.
2. Addition — If $u < v$, then $u + w < v + w$.
   If $u < v$ and $w < z$, the $u + w < v + z$.
3. Multiplication — If $u < v$ and $c > 0$, the $uc < vc$.
   If $u < v$ and $c < 0$, then $uc > vc$.

The above properties are true if < is replace by $\leq$. There are similar properties for > and $\geq$.

(Do examples of solving equations)